Indigenous Japanese Mathematics, Wasan

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Historical Background and Prehistory

Every civilization in history has its own fate: awakening, growing up, flourishing, and declining. Indeed, even the Greek civilization had once declined, and, after having sunk into oblivion, the Renaissance literally "revived" it. A tradition which started in antiquity and has persisted up to the present, if any, would be the Chinese tradition, whose most noteworthy features include several philosophical ideas and the invention of paper and Chinese characters (*Kanji*).

The tradition of Japanese civilization, even if it is not so long and profound as that of the Chinese, has similar phases at any rate. It started fairly early, even though it does not possess any original religion or philosophy. As a matter of fact, the Japanese civilization developed as a satellite of China, but this does not mean that the Japanese simply imitated the Chinese. After having adopted and assimilated it, the Japanese began to advance and refine it in their own way, of which one characteristic is their particular sense of beauty, and another is perhaps a lack of original thinkers of the first order.

Today the opinion seems to be widely accepted that a driving force of the rapid progress in modern Japan is found in the Meiji Restoration (1868) or, at the latest, in the Americanization after the defeat (1945). However, this opinion is not appropriate, because what supported that progress is a large amount of the cultural accumulations in history, particularly in the Edo period (under the Tokugawa shogunate; 1603–1867). In fact, at the beginning of the 6th century, books including Buddhist sutras (written in Kanji) were officially imported into Japan via Korea. It was mainly Korean immigrants who played leading roles in ancient Japan. It is important to bear in mind that China and Korea were the first benefactors of Japanese culture.

The Japanese culture begun to flourish from the beginning of the 7th century, including the remarkable invention of their phonograms (7–8th centuries; *Kata-kana*, *Abridged Chronological Table of Cultural Events*

Awakening preiod	First official contacts with China (1st-7th centuries); National organization (7th centuly); Horyuji temple (7th century, the oldest wooden building in the world).
Nara period (710–793)	Nara Capital; The University established, where Chinese mathematical texts were adopted, but did not take root; Buddhism flourished; National histories edited; *Manyoshu* anthology comiled.
Heian period (794–1192)	Heian Capital (Kyoto) (794); Studies of Buddism; First national isolation (802–12th century); Development of *Kana* literature, including *Genji-monogatari* (about 1007)
Samurai goverments' period (1192–1573)	In spite of militaly goverment, the study of the calssics, includeing mathematics, continued in Buddhist temples in Kyoto.
Civil war period (1603–1867)	Reopenign of civil contacts with china and Korea, and new mathematical texts imported (sources of Wasan); First contact with European nations.
Edo preiod (1603–1867)	Feudal system employed but not totally despotic; Second national isolation (1639–1854), when only nationals from China and Holland could visit; Peaceful preiod in which Wasan started and flourished; The so-called Japanese classical cultures took root.
Modern period (1868–present)	(a) Meiji Restoration; Europeanism; The establishment of the New Goverment, where Western mathematics was officially adopted; End of Wasan; Confrontation of democracy vs. militarism. (b) First World War; Growth of militarism; Aggression in China; Second World War. (c) Defeat (1945); Disamament; Americanization; Economic and technological prosperity.

Hira-kana, etc.), which consist of the modification of Chinese ideograms, Kanji[1]. And another first-class original creation appeared in the 17th–18th centuries. This is *Wasan*, indigenous Japanese mathematics, which keenly represents the Japanese originality. However, contrary to the above mentioned features of Japanese history, Wasan declined towards the end of the 19th century after about 200 years of development. But a new mathematical tradition is reviving in Japan.[2]

General Survey of Wasan

1) The invention of *kana* is really original to Japan. It accommodated a radical difference found in the syntax of the Japanese language and the Chinese; the former is an agglutinate language, while the latter is an isolating one. This invention has been considered a first-class creation by the Japanese. But a document was discovered at the end of 2000 in Korea, which raised the question: who was the real inventor of *kana*, the Japanese or the Koreans? For the time being, so long as the existent documents are concerned, the Japanese seems to be the original and real inventors. But it would be fair to say that this question is open at present.

2) Our aim is not to inform readers of the present day mathematical activities in Japan, but one thing should be noted as a typical example. In 1954, the Mathematical Society of Japan edited *lwanami Sugaku Jiten* (Encyclopedic Dictionary of Mathematics). The 2nd edition (1968) and 3rd edition (1985) were largely revised and enlarged. The 2nd edition was translated into English in two volumes and published by MIT Press in 1977, and the (Japanese) 3rd edition was again translated in four volumes and published as a 2nd edition by MIT Press in 1987.

Wasan is the most original cultural creation that the Japanese have ever realized. But, of course, it was not a *creatio ex nihilo*. Japan has acquired mathematical knowledge from abroad over three periods: Classical Chinese mathematics in the 6th to 9th centuries, contemporary Chinese mathematics in the 15th and 16th centuries, and European mathematics since the middle of the 19th century.

Wasan was created on the basis of the second acquisition and, after the third acquisition, it rapidly declined. *Wa* means "Japan" or "Japanese," and *san* (*suan* in Chinese) means "calculation" or "arithmetic." At that time, this learning was called by many names in Japanese, among them *san*, *sampo* or *sanho* (*suanfa* in Chinese), *sangaku* (*suanxue* in Chinese) and *sanjutu* (*suanshu*), where *ho* (*fa*) means "rule" or "method," *gaku* (*xue*) means "learning" or "science," and *jutsu* (*shu*) means "art" or "technique." Now, the word "mathematics" is derived from the Greek *mathémata*, which originally meant something learned, or learnable knowledge in general. Concerning this transition, there is a long and intricate history in the West from Pythagoras up to the present. It exerted various profound influences upon Western culture, including philosophy and the natural sciences. On the other hand, although *san* has its own history, it had little influence upon any logico-philosophical thinking in Japan.[3] In addition, Wasan had an aesthetic character, more or less connected with the fine arts. So, it may be questionable whether *san* can simply be translated as "mathematics" without any qualification, but it is beyond doubt, too, that most aspects of Wasan were consistent with Western mathematics. Therefore, our main concern will be summarized in the following two points.

1) To what extent is Wasan similar to Western mathematics?

2) In what respects are they differentiated?

Wasan, constructed independently of Western mathematics, was based on a perfect understanding of Chinese mathematics. However, curious as it may appear for Europeans, Wasan left no trace of deductive reasoning in the Euclidean sense, nor of philosophico-mathematical reflections on the universe, through which Europeans initiated the modern natural sciences. The only concern of Wasanists was to obtain

3) From a certain point of view, the ancient doctrine of *Yi-king* (Book of Changes), which was developed by scholars in the Sung Dynasty (960-1279) and inherited by Japan, can be regarded as a philosophy of number, accordingly of mathematics. Of course, this philosophy was "logically" constructed. We believe, however, that Yi-king philosophy and Western philosophy are completely different.

elegant results by means of numerical law or of geometrical construction, and for this very purpose they did not hesitate to make enormous calculations. Thus, we can say that many Wasanists were not wholly men of mathematics in the European sense, but in a sense men of the fine arts.[4)]

The representative Wasanists are Seki Takakazu (or Kowa) (1640/42–1708)[5)], Takebe Katahiro (1664-1739), Kurushima Yoshihiro (died in 1757) and some others cited below. It is said that almost all the basic ideas were laid down by these three men, especially by Seki. Nowadays he is regarded by mathematicians worldwide as a first-class mathematician.

All the advancements of Wasan took place in the Edo era, when Japan kept her gates tightly closed to the West for political and religious reasons. In 1868 the Emperor was restored, and the new government adopted Western mathematics in the national educational system. The history of Wasan ended at this time. However, this decision proved after all to be very wise; because, in spite of all its originality and excellence, Wasan would not have had the potentiality to raise Japan to her present level of scientific achievement.

The Origins and Rise of Wasan

In the official catalogue, *Nipponkoku Kenzaisho Mokuroku* (List of Chinese books existent in Japan; compiled in 891-894), more than 1,500 books are listed, including many Chinese mathematical texts. Among them, *Jiuzhang Suanshu* (Arithmetic in Nine Chapters, compiled in the 2nd–3rd centuries; *Kyusho Sanjutsu* in Japanese) is the most famous. There is still another more important book in the list, *Zhuishu*, which will be cited in the sequel. In any case, the Japanese at that time could not develop their own mathematics.

This situation had gradually changed by the 17th century with the help of two mathematical books imported via Korea: Zhu Shiji's *Suanxue Qimeng* (Introduction

4) An aesthetic tendency can be observed, however, among the first-class European mathematicians such as Newton. Generally speaking, the mathematical truth has a character which tends to explore beauty.

5) Except for his articles and books (less than 20 volumes), little is known about Seki's career, because he was not a high-ranked Samurai and his family was extinguished soon after his death. His birth year is obscure and thus his birth place is also not explored: if he was born in 1640, the place would have been near Edo, but if his birth year was 1642, then Fujioka town in Gunma Prefecture seems likely. It was an historian in the late 19th century who forged the date to be 1642, for the purpose of identifying the date with that of Newton! Although it might be trivial from a scientific point of view, this fact could show the low social status of Wasan and the Wasanists at that time, compared with that of European mathematics and mathematicians.

to Mathematics, 1299; in Japanese, *Sangaku Keimo*) and Cheng Dawei's *Suanfa Tongzong* (Account of Arithmetic, 1592; *Sampo Toso* in Japanese). The latter is an introduction to elementary mathematics and a manual for the abacus. On this basis, Yoshida Mitsuyoshi (1598-1672) compiled an excellent book, *Jinkoki* (Book of the Smallest and Largest Numbers, 1627). This became a best-seller throughout the Edo period, and not only laid the basis for higher mathematics but also played a prominent role in popularizing elementary mathematics among the people.

What directly contributed to the rise of Wasan was *Suanxue Qimeng*, a textbook of higher instrumental algebra (*tianyuan shu, tengenjutsu* in Japanese), which made use of the abacus and calculating chips to solve equations numerically. But, by the time of its arrival in Japan, nobody in Japan or in China could have understood its contents, because it presented only various rules but gave no explanations; it was something like a concise manual of an electronic computer. A civil war in China completely destroyed its scientific tradition. The Wasanists were forced to understand it. At the time when their task was completed, Wasan had already surpassed its mother source. The process of this development can be observed in the following series of books: *Kokon Sampoki* (1671, Lectures on Mathematics in Ancient and Modern times) by Sawaguchi Kazuyuki (dates obscure), *Sampo Ketsugi Sho* (1659, revised in 1684; Short Account of Clarifying Mathematical Problems) by Isomura Yoshinori (died in 1710), and *Sampo Futsudankai* (1673; Never Hesitate to Improve Mathematical Investigation) by Murase Yoshimasu (dates obscure), a disciple of Isomura.

Murase also invented a method (similar to successive approximation) for giving a numerical approximate solution of equations of the third degree.[6)] The method of this sort later played an crucial role for treating numerical problems and largely contributed to the advancement of Wasan, because many problems in Wasan were almost always given and solved in numerical form, and these procedures often played an heuristic role in further studies. In fact, the very source of almost all the original

6) When a wooden frame is constructed by four congruent square pillars, from its volume V and external side h, to find out the side of square x:

$$V = 4x^2(h - x), \text{i.e.} 4x^3 - 4hx^2 + V = 0.$$

results in Wasan, theorems or theories, is often found in numerical examinations.

Concerning geometry, many problems were also introduced from China. For example, the Pythagorean theorem was already known as early as *Jiuzhang Suanshu*, but as a numerical problem without proof. The first "proof" was given in the form of a puzzle in *Sampo Futsudankai*.

It is Seki who put a finishing touch to scrutinizing that enigmatic Chinese instrumental algebra and on this basis he invented a notational algebra, called *Endan Jutsu*. And this is the very starting point of Wasan.

Flourishing Epoch of Wasan

The establishment of *Endan Jutsu* was the driving force of Wasan. In fact, it works as a record of process in resolving equations with which one can objectively follow the forerunners' line of thinking without hand-in-hand teaching. It also revealed a crucial structure for solving methods. However, we must not overestimate the value of algebraic operation of *Endan Jutsu*. In addition to it, without a tradition of Euclidean deductive mathematics, his algebra would have never reached the level of Descartes' operational algebra, and only remained at the level of Coss' algebra (Italian algebra in the 16th century). While Coss' algebra was a prescription to record formulas in abbreviated sentences,[7)] the Cartesian algebra was not only notational but also operational in the sense that one could execute operations in symbolic notations.

With the aid of this method, Seki explored various fields of mathematics. In his youth he invented an original theory of determinants. It was not completed but later it was improved by his successor Matsunaga. Even though his work preceded Leibniz' theory of determinants, Seki would not be able to declare his priority because of its incompleteness. However, it is important to note his ability of detecting problems from unknown chaos as well as that of promoting them up to a mathematical theory.

Seki's theory of equations, being advanced with his determinant theory, is also original, and perhaps more important from a mathematical point of view. Since it is not our aim to enter into mathematical details, however, we will just enumerate

7) Algebra in which an equation and its solution was written in an abbreviated form. An unknown quantity was often represented by *"tres"* (Latin), where *"Coss"* (in Italian; "thing," in English) came from *"res."*

some of his acquisitions without explanation, which will help us to see his originality and variety. For the purpose of concreteness, however, we shall discuss a history of the theory of circles (*Enri*), separately in the next paragraph (*En* is "circle" and *ri* is "reason" or "truth").

A theory of those numbers which have the form $(1^p+2^p+3^p+\cdots+n^p)$, where p and n are natural numbersr[8)]; Horner's method and Newton's method for numerically solving equations of higher degree, (the former was widely known in Japan far earlier than in Europe, while the latter was a sub-production of Newton's calculus); many geometrical problems such as conic sections and Archimedes' spiral; many theories picked up from mathematical recreations, including the magic square. Some of these contributions preceded European scholars, and some others were established in totally proper ways. It is for this reason that we call him a first-class mathematician in the world history of mathematics.

Seki and his disciple Takebe Katahiro studied the Calendar and Astronomy. These are simple applications of mathematics. However, concerning the relationship between mathematics and the real world, even such simple applications were seldom studied by their successors, not to mention applications to the physical world. It seems strange for us living in a modern age, and we might conclude that this shows the limits of Wasan. However, it is important for us to note here that this is the most revolutionary point in history. Applying mathematics for solving the riddles of the cosmos is not self-evident. Today the idea that mathematics can clarify these riddles seems banal, but truly, this was one of the driving forces behind the so-called scientific revolution. So, even if the Wasanists could not arrive at such an idea, it would not be their defect. Rather, it can show the height of European cultural tradition.

Takebe Katahiro, together with his brother Takebe Kataakira (1661–1739), under the editorship of Seki, made a compilation of all the mathematical acquisitions of their predecessors and themselves in 20 volumes: *Taisei Sankyo* (Large Collection of Mathematics) in 1683–1710.

Kurushima Yoshihiro also developed many subjects. For example, he improved

8) This theory is part of the theory of Bernoulli numbers. Seki's theory was limited within the number theory and would be utilized by his successors to get definite integrals.

Seki's theory of determinants and solved some maximum-minimum problems.[9] According to a simple criterion of priority, some of his results preceded Euler's and others'. But we must take notice of a difference in their methods: Euler's was systematically constructed within the frame of Leibnizian calculus, while Kurushima's was not systematic. In addition, as a result of his negligent character, most of his papers have been lost for ever.

Matsunaga Yoshisuke (1639?–1744), a friend of Kurushima, established many formulas, equivalent to the corresponding Taylor series.[10] But more noteworthy are his ideas on mathematics. According to his letter to Kurushima, Matsunaga intended to compile a systematic textbook of Wasan, and asked Kurushima to cooperate with him on the project. He wrote as follows:

> *I have kept in mind an intention for many years and will realize it when I shall be fifty years old. But, once I become fifty, I was miserably aware that it is too late as a result of a decline of my health and mental ability. Now, looking around myself, there is no one but you, who would be able to carry it out. Why don't you dare to try to realize it, letting aside very artificial problems, elegant results or cunning methods? ...*

However, Kurushima did not reply to Matsunaga's hope, and in the end his project was not realized. Kurushima, as an eminent technician, was hardly capable of such systematic and theoretical thinking. This was perhaps the beginning of the decline of Wasan.

Concerning studies of π

Solid evidence for the independence of Wasan from Western mathematics is found in the studies of π. We will examine this point in detail below.

As is often the case with oriental mathematical texts, the Chinese texts which they could refer to contained only approximations such as $\sqrt{10}$, 3.14, without providing any indication of how the values were found out. The Wasanists' first efforts were devoted to reproducing these results from classical texts.

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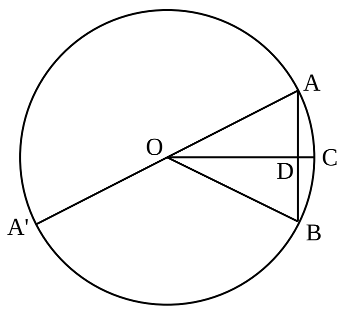
9) The calculus supplied a powerful method for maximum-minimum problems, but, without the calculus, Kurushima largely explored the same problems.

10) When a function f can be represented as a power series, we call it a Taylor series of f, e.g. $\sin x = x - x^3/3! + x^5/5! - x^7/7! + \cdots\cdots$, where x is an angle measured by "radian."

In 1663 Muramatsu Sigekiyo (dates obscure) calculated the circumference of an inscribed regular polygon with $2^{15}(= 32768)$ and defined π as the concrete number of 3.14, in spite of the fact that his more accurate result $3.14159264\cdots\cdots$ (correct value being $3.14159265\cdots\cdots$) had already been developed. In 1672 Murase made the same calculation for a polygon of $2^{17}(= 131072)$ sides, and gave π as 3.1415.

It is curious that neither Muramatsu nor Murase explained how they came up with their approximations. A possible explanation is that Japanese scholars in various fields often too highly esteemed the Chinese classics and regarded them as canons of their studies. The Wasanists also adopted the values discovered in the Chinese classics as a criterion.

These studies of π were continued by subsequent generations, but none of them, including Seki and Takebe, ever calculated the corresponding circumscribed polygons or the like, as Archimedes did. So, strictly speaking, those values which they developed had scarcely logical foundations. However, only one exception existed in Osaka, where Kamata Yoshikiyo (1678–1744) estimated the value of π, and defined the upper and lower limits for polygons with $2^{44}(\fallingdotseq 175$million millions) sides. But Kamata had little connection with Seki's school in Edo (now Tokyo), and his method was never taken up elsewhere. This can be regarded as a theoretical limitation of Seki's school and of Wasan.



Seki made remarkable progress in this research. He disposed in order each circumference s_n of 2^n sides-polygon (inscribed in a unit circle), and made a (finite) series: $s_2, s_3, \cdots, s_{17}$ (s_2 corresponds to square, and s_{17} to 131072-sided polygon) – what astonishing perseverance! Then he discerned that this series could be approximately equal to an (infinite) geometric progression. Having taken the sum (or limit) of it, he declared that π is "a little less than 3.1415926539" (the true value is $3.1415926535\cdots\cdots$).

Takebe largely revised Seki's method, and finally arrived at a far better approximate:

$$3.14159\,26535\,89793\,23846\,26433\,83279\,50288\,419712 + \epsilon,$$

(where 40 places are correct).

In order to obtain this, he calculated $(s/2)^2$ (where s is a small arc AC) using Seki's method repeatedly, and found out the value $(1/10^4)\cdot(1.00000\,03333\,33511\,1\cdots\cdots)$. Taking this value as a key, he tried to form an approximate polynomial, of which the given value is c/R, (where R is a diameter and c is a small segment between the arc and the corresponding chord CD). Since $1.00000\,03333\,33511\,1\cdots\cdots$ is nearly equal to $1+(1/3)\cdot(1/10^6)$, the rest would be determined in a similar way, and he succeeded in realizing it through a very ingenious calculation. The following is his last result, for $R = 10$ and $c = 10^{-5}$.[11)]

$$\left(\frac{s}{2}\right)^2 = \frac{1}{10^4}\left[1 + \left(\frac{2^2}{3\times 4}\times\frac{1}{10^6}\right) + \left(\frac{4^2}{5\times 6}\times\frac{1}{10^{6\times 2}}\right) + \left(\frac{6^2}{7\times 8}\times\frac{1}{6^{6\times 3}}\right) + \cdots\cdots + \left(\frac{12^2}{13\times 14}\times\frac{1}{10^{6\times 6}}\right)\right]$$

Although this formula is finite because of the regularity of the coefficients,

$$\left(\frac{2^2}{3\times 4}\right),\ \left(\frac{4^2}{5\times 6}\right),\ \left(\frac{6^2}{7\times 8}\right),\ \left(\frac{8^2}{9\times 10}\right),\ \left(\frac{10^2}{11\times 12}\right),\ \left(\frac{12^2}{13\times 14}\right)$$

we can easily prolong this formula *ad infinitum.*

This series would prove to be equivalent to the Taylor series of $\sin^{-1} x$,[12)] published 15 years later by Euler. It is true that Takebe's method was not so theoretical as Euler's, but remained on a numerical level. Euler's method was based on the *Analyse Infinitesimale* of Leipniz, a systematic theory of calculus, and had a far wider field of application. But without doubt Takebe's perseverance and penetration are astonishing and his creativity is worthy of admiration. We believe Takebe, as well as Seki, can be ranked with Euler and Newton, at least in this respect.

Takebe's work is compiled in his *Tetsujutsu Sankyo* (1722). Literally, *Tetsujutsu* means "art of knitting," and substantially, "art of repetition or mathematical induction." Probably he adopted this title from a great Chinese classic, *Zhuishu* (in Japanese, *Tetsujutsu*) written by Zu Chongzhi (c.a. 429–500), an Archimedes in

11) In order to obtain the formula of $(s/2)^2$, first he had to get the series of coefficients, 1/3, 8/15, 9/14, 32/45, 25/33, 72/91. It goes without saying that this work needed an ingenious device, but he did not stop there. By way of multiplying 2^2 to the nominator and denominator of the 1st, 3rd and 5th fractions, and by way of multiplying 2 to those of the 2nd, 4th and 6th fractions, he got the regular and beautiful forms, such as $2^2/(3\cdot 4)$. What a marvelous penetration!

12) When $y = \sin x$, an inverse function which defines x by y is called the "arc sine of y"; in symbols, $x = \sin^{-1} y$.

China. Although it is a fact that this work was registered in the above-cited *Nipponkoku Kenzaisho Mokuroku*, it had been lost shortly after its publication in China, and after the 10th century it was lost in Japan, too. Thus, all that was available for Wasanists were those results of Zu: $3.1415927 > \pi > 3.1415926$, and two approximates 22/7 and 355/113. We can find these records in the official history of the Sui Dynasty (636). So, there is no room for Takebe to have referred to Zu's method.

Tetsujutsu Sankyo is perhaps the only book on methodology and philosophical reflections of mathematics in the history of Wasan. Takebe's "philosophy of quality" is particularly interesting. First, he distinguished two kinds of quality among mathematicians: the analytical and the intuitive. The former characterizes those, like himself, who never hesitate to make enormous calculations; and the latter characterizes those who prefer to be an "armchair detective."[13] Next, he classified three kinds of infinity in mathematics on the basis of the decimal system,: (1) infinity in number like infinite decimals, (2) infinity in operation, including those operations which produce an infinity in number for a finite number/numbers (e.g. division, square root), whereas those which always produce a finite number and for finite number(s) are called finite in operation (e.g. addition, multiplication), and (3) infinity in quality, whose value will never be determined as a finite number for any given finite number(s) nor by any finite operation. According to him, the reason why he succeeded in the study of π by means of an infinite series, while Seki did not, is that the circle has a character of infinity in quality, and that Seki's intuitive quality as a mathematician did not fit the quality of the problem. He then concludes that, when a quality of mathematician meets "by chance or by coincidence" with a problem of the same quality, the problem will be solved, otherwise it would be hard to resolve.

Observing such a conscious usage of infinite series, it would be safe to say that Takebe had a clearer idea of it than Seki, to say nothing of other Wasanists, even if it was not so perfect as in the present mathematics. When he penetrated the possibility of prolonging the series, he must have believed in a harmony in the mathematical world, believing in the same regularity in this world. However, it seems that even

13) This remark of his may be associated with Henri Poincaré's classification of types of mathematician: "logician by analytical calculation" and "geometrician by inner discernment" (Poincaré, *Science et Methode*, 1908).

Takebe never thought of the harmony between the (internal) mathematical world and the external physical world. This is perhaps the most fundamental philosophical difference between Wasan and Western mathematics.

Takebe's phrase "encounter by chance or by coincidence"is also significant. It is not only unique but also thought-provoking for a future philosophy of mathematics. In fact, in taking this aspect of harmony into account, Takebe's comments on the "quality of mathematician" and "encounter by chance" seemingly suggest to us a significant meaning, because they might allude the possibility of a new approach to a philosophy of mathematics which stands not only on the harmony existent among mathematics and physics, but also another harmony between the subjectiveness and the objectiveness.

Decline of Wasan and New Contact with the West

After Matsunaga and Kurushima, the activity of Wasan gradually declined. Although numerical investigations of infinite series were still being investigated, a new trend of Wasan began to diverge from essential problems, pursuing something colored with simple curiosity or aesthetic beauty. Of course, there still existed a few good Wasanists. In the late 18th century, Ajima Naonobu (1739-1798) obtained some theorems which had not yet been known in Europe by that time. He developed them on the basis of an imported table of logarithms. He composed, too, a table of definite integrals of a sort. Wada Nei (1787–1840), the eminent successor of Ajima, composed a further table of definite integrals, but he led a miserable life and died in the neediest circumstances. His many contributions were lost, except for the papers published under his pupils' names – a symbolic episode of the decline of Wasan.

Even during the time of the second national isolation, there still remained the channels via China and Netherlands, through which some knowledge of the Western world could be learned. After 1720, imports of non-religious European books were permitted. By the end of that period, some people had a fair knowledge of the world situation, including Western sciences. Indeed, there appeared an abridged translation of Aristotle's *Metaphysica* or of Newton's *Principia* (Dutch versions translated). However, those who introduced Western technology and science were mainly a group of *"yogakusha,"* (scholars of Western learning), and, curious to say, Wasanists were not always interested in these subjects, nor in Western mathematics. When they saw the Chinese translation of the Euclidean *Elements*, they could not recognize the importance or the value of it, having misjudged it as too elementary, probably compared with the complicated figures of Wasan.

Nowadays mathematical activities in Japan have hardly any connection with Wasan. In this respect, there are some scholars who speak of a possible negative influence of Wasan. In fact, until quite recently there was a tendency among many Japanese mathematicians to study pure mathematics and other mathematical sciences separately, and only a few philosophical reflections were attempted. It is only very recently that this tendency has begun to fade away. (see note 2))

Additional Comments on Independence and Originality

The independence and originality of Wasan can be shown in three ways: (a) the period when Wasan developed most rapidly was the period of national isolation; (b) one can evaluate the way in which Wasan advanced, as is seen in the case of π; and (c) some results were obtained in original ways, many of which were numerical.[14)]

Some Japanese historians advocated that the infinitesimal calculus was invented by Seki, Takebe and a few others in the form of *Enri*, but this claim was an exaggeration and has now been abandoned. Despite its originality, the achievements of Wasan form a group of fragmentary results, as outlined in this article. Even some fundamental concepts in analysis, including the variable, the function and differentiation, never appeared, to say nothing of the fundamental theorem of calculus, that is the inverse relationship between integration and differentiation, and the mathematical natural philosophy developed by Descartes, Newton, Leibniz and others.

However, one must here take into account the second national isolation. Most of Wasan's results were developed in a world of isolated cultural tradition. Especially, it is worth noting that it was separated from the tradition of Greek mathematics.

14) Truly, a numerical examination may not always characterize the Wasanists' activity, because in Europe, especially in the 17–18th centuries, many eminent mathematicians adopted it at least as an heuristic method, including Newton and Euler. Then, should we emphasize again the weight of tradition: whether the Greek tradition had already been accepted or not yet?

So, for all its weaknesses, Wasan should be appreciated at least as an unusual phenomenon in the world history of mathematics.

It must be noted, too, that all these points are closely related to the fundamental problem of the comparative history of mathematics, or rather that of the comparative history of human civilization. The independence of Wasan can be considered as plausible evidence of the universality of mathematics, on the one hand, and on the other hand, it is questionable to decide whether an objective comparison between Wasan and European mathematics is possible. On what basis should an impartial and objective comparison be made? Or rather, is it possible to establish such a basis? Just as the tradition of Western culture has its own bias, so does the Japanese. This kind of comment, pedantic as it may appear, would produce some fruitful results not only for the comparative history of mathematics, but also for the philosophy of math ematics. (see Murata, T. 1981)

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